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$n\nu$-Lindelöfness

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ABSTRACT

Our work aims to study nearly ν-Lindelöf (briefly. $n\nu$-Lindelöf) space in generalized topological spaces. Moreover, some mappings and decompositions of continuity are studied. The main result that we obtained on is the effect of (δ, δ')-continuous function on $n\nu$-Lindelöf space.

Keywords: ν-Lindelöf $\mathcal{GTS}$, $n\nu$-Lindelöf $\mathcal{GTS}$ and G-semiregular $\mathcal{GTS}$.

1. Introduction

The study of generalized topological spaces was first initiated byCsàszàr (1997), and therefore a lot of authors have been achieved to generalize the topological notions to generalized topological surroundings. In literature, there are several generalization of the notion of regular sets, and these are studied separately for different reasons and purposes. Csàszàr (2008) defined ν-regular open (resp. ν-regular closed) sets. Sarsak (2012) introduced and studied ν-compact (resp. ν-Lindelöf) sets in generalized topological spaces. After that, Arar (2014) gave the corresponding definitions of paracompact spaces in generalized topological spaces. Kiliçman and Abuage (2015) studied some spaces generated by ν-regular sets, namely; almost G-regular and G-semiregular spaces in generalized topological spaces. In this present work we will be define other generalization of ν-Lindelöf in generalized topological spaces namely; nearly ν-Lindelöf (briefly. $n\nu$-Lindelöf). In third section, we shall introduce the concepts of $n\nu$-Lindelöf generalized topological spaces, and obtain on some results. Furthermore, the relation between $n\nu$-Lindelöf, ν-Lindelöf have been given, some characterizations of the concept of $a\nu$-Lindelöf subspaces and subsets are investigated. The primary result is that the $n\nu$-Lindelöf generalized topological space is not a ν-hereditary property. In forth section, we shall introduce the effect of some mappings and decompositions are studied. The main result of our study is that a (δ, δ')-continuous image of $n\nu$-Lindelöf generalized topological space is $n\nu$-Lindelöf.

Suppose a non-empty set $\mathcal{X}_\mathcal{G}$, $P(\mathcal{X}_\mathcal{G})$ denotes the power set of $\mathcal{X}_\mathcal{G}$ and ν be a non-empty family of $P(\mathcal{X}_\mathcal{G})$. The symbol ν implies a generalized topology (briefly. $\mathcal{GT}$) on $\mathcal{X}_\mathcal{G}$ Csàszàr (2002) if the empty set $\emptyset \in \nu$ and $\mathcal{U}_\gamma \in \nu$ where $\gamma \in \Omega$ implies $\bigcup_{\gamma\in\Omega} \mathcal{U}_\gamma \in \nu$. The pair $(\mathcal{X}_\mathcal{G}, \nu)$ is called generalized topological space (briefly. $\mathcal{GTS}$) and we always denote it by $\mathcal{GTS}$ $(\mathcal{X}_\mathcal{G}, \nu)$ or $\mathcal{X}_\mathcal{G}$. Each element of $\mathcal{GT}$ ν is said to be ν-open set and the complement of ν-open set is called ν-closed set. Let $\mathcal{A}$ be a subset of a $\mathcal{GTS}$ $(\mathcal{X}_\mathcal{G}, \nu)$, then $i_\nu(\mathcal{A})$ (resp. $c_\nu(\mathcal{A})$) denotes the union of all ν-open sets contained in $\mathcal{A}$ (resp. denotes the intersection of all $\nu-$closed sets containing in $\mathcal{A}$), and $\mathcal{X}_\mathcal{G}\backslash\mathcal{A}$ denotes the complement of $\mathcal{A}$. Moreover, $\mathcal{A}$ is said to be ν-regular open (resp. ν-regular closed) iff $\mathcal{A} = i_\nu c_\nu(\mathcal{A})$ (resp. $\mathcal{A} = c_\nu i_\nu(\mathcal{A})$) Csàszàr (2008). If a set $\mathcal{X}_\mathcal{G} \in \nu$, then a $\mathcal{GTS}$ $(\mathcal{X}_\mathcal{G}, \nu)$ is called ν-space Noiri (2006), and will be denoted by a ν-space $(\mathcal{X}_\mathcal{G}, \nu)$ or a ν-space $\mathcal{X}_\mathcal{G}$. $\mathcal{X}_\mathcal{G}$ is said to be quasi-topological space Csàszàr (2006), if the finite intersection of ν-open sets of ν belongs to ν and denoted by $\mathcal{QTS}$. If $\mathcal{B} \subseteq P(\mathcal{X}_\mathcal{G})$ and $\emptyset \in \mathcal{B}$. Then $\mathcal{B}$ is called a ν-base Csàszàr (2004) for ν if $\{\cup\mathcal{B}' : \mathcal{B}' \subseteq \mathcal{B}\} = \nu$, and we say that ν is generated by $\mathcal{B}$. A $\mathcal{GT}$ ν generated by ν-regular open sets of a $\mathcal{GTS}$ $(\mathcal{X}_\mathcal{G}, \nu)$ is said to be ν-semiregularization Kiliçman and Abuage (2015) of $\mathcal{X}_\mathcal{G}$, denoted by $\mathcal{GTS}$

$(\mathcal{X}_{\mathcal{G}}, \nu_\delta)$. $\mathcal{X}_{\mathcal{G}}$ is said to be G-regular Min (2010a) if for each $t \in \Lambda_\nu$ and each ν-closed set $\mathcal{F}$ with $t \notin \mathcal{F}$, there are disjoint ν-open sets $\mathcal{U}$ and $\mathcal{V}$ such that $t \in \mathcal{U}$ and $\mathcal{F} \cap \Lambda_\nu \subseteq \mathcal{V}$, where Λ_ν is the union of all ν-open set in $\mathcal{X}_{\mathcal{G}}$. A $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is called submaximal Ekici (2012) if every ν-dense set of $\mathcal{X}_{\mathcal{G}}$ is ν-open, and is said to be ν-extremally disconnected Csàszàr (2004) if the ν-closure of every ν-open set is ν-open. Moreover, a subset $\mathcal{A}$ of a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is called ν-clopen if it is both ν-open and ν-closed subset.

Theorem 1.1. Kiliçman and Abuage (2015)

(a) *A $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is G-semiregular if for each point $t \in \Lambda_\nu$ and each ν-open set $\mathcal{U}$ containing t, there exists ν-open set $\mathcal{V}$ such that $t \in \mathcal{V} \subseteq i_\nu c_\nu(\mathcal{V}) \cap \Lambda_\nu \subseteq \mathcal{U}$.*

(b) *A $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is almost G-regular if each point $t \in \Lambda\nu$ and each ν-regular open set $\mathcal{U}$ containing t, there exists ν-open set $\mathcal{V}$ such that $t \in \mathcal{V} \subseteq c_\nu \mathcal{V} \cap \Lambda_\nu \subseteq \mathcal{U}$.*

Definition 1.1. Arar (2014) *Let a ν-space $(\mathcal{X}_{\mathcal{G}}, \nu)$, then*

1. *A family ξ of subsets of $\mathcal{X}_{\mathcal{G}}$ is called ν-locally finite if for each $t \in \mathcal{X}_{\mathcal{G}}$ there is ν-open set $\mathcal{U}$ containing t such that $\mathcal{U}$ intersects at most finitely many elements of ξ.*

2. *Let a ν-open cover $\xi = \{\mathcal{V}_\gamma : \gamma \in \Omega\}$ of $\mathcal{X}_{\mathcal{G}}$, a collection $\eta = \{\mathcal{U}_\alpha : \alpha \in \Gamma\}$ of ν-open subsets of $\mathcal{X}_{\mathcal{G}}$ is said to be a ν-open refinement of ξ if η is cover of $\mathcal{X}_{\mathcal{G}}$ and each $\mathcal{U} \in \eta$ there is $\mathcal{V} \in \xi$ such that $\mathcal{U} \subseteq \mathcal{V}$.*

Definition 1.2. Abuage et al.. *A $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is said to be:*

1. *$n\nu$-compact if each ν-open cover $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ of Λ_ν admits a finite sub-collection $\{\mathcal{U}_{\gamma_k} : k = 1, 2, ..., \mathfrak{n}\}$ such that*

$$\Lambda_\nu = \bigcup_{k=1}^{\mathfrak{n}} (i_\nu(c_\nu(G_{\gamma_k}))).$$

   i.e. each ν-regular open cover admits a finite sub-collection.

2. *$n\nu$-paracompact space if each ν-regular open cover of Λ_ν has a ν-open locally finite refinement.*

2. $n\nu$-Lindelöf $\mathcal{GTS}$ and Subspaces

Sarsak (2012), defined a ν-Lindelofness, since a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is called ν-Lindelöf if each ν-open cover of Λ_ν admits a countable sub-collection.

Definition 2.1. *A $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is called nearly ν-Lindelöf (briefly. $n\nu$-lindelöf) if each ν-open cover $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ of Λ_ν has a countable sub-collection $\{\mathcal{U}_{\gamma_{\mathfrak{n}}} : \mathfrak{n} \in \mathfrak{N}\}$ such that*

$$\Lambda_\nu = \bigcup_{\mathfrak{n}\in\mathfrak{N}} (i_\nu(c_\nu(\mathcal{U}_{\gamma_{\mathfrak{n}}}))).$$

That means every ν-regular open cover of Λ_ν has a countable sub-collection.

Remark 2.1. *Clearly, that every $n\nu$-compact space is $n\nu$-Lindelöf but the converse in general is not true as in Example 2.6.* Arar (2014). *Since a collection $\{\{1, \mathfrak{n}\} : \mathfrak{n} \in \mathfrak{N}, n \geq 2\}$ is a ν-regular open cover of ν-space $(\mathfrak{N}, \nu(\beta))$ with no finite sub-collection. Further, every ν-Lindelöf space is $n\nu$-Lindelöf but in general, the other hand not necessary be true as in the Example below:*

Example 2.1. *Let $\mathcal{X}_{\mathcal{G}} = \{a, b, c, ...\}$ be infinite set and $\beta = \{\{a, t\} : t \in \mathcal{X}_{\mathcal{G}}, a \neq t\}$. If the $\mathcal{GT}$ $\nu(\beta)$ generated on $\mathcal{X}_{\mathcal{G}}$ by the ν-base β. Thus only $\{\mathcal{X}_{\mathcal{G}}\}$ is ν-regular open cover of itself, so a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu(\beta))$ is $n\nu$-Lindelöf but it is not ν-Lindelöf, since $\{\{a, t\} : t \in \mathcal{X}_{\mathcal{G}}, a \neq t\}$ is a ν-open cover of $\mathcal{X}_{\mathcal{G}}$ with no countable sub-collection.*

Definition 2.2. *Let a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$. A sub-collection $\{\mathcal{S}_\gamma : \gamma \in \Omega\}$ of $P(\mathcal{X}_{\mathcal{G}})$ is said to satisfy a countable intersection property if for every countable sub-collection $\{\mathcal{S}_{\mathfrak{n}} : \mathfrak{n} \in \mathfrak{N}\}$ of $\mathcal{S}$, the intersection $\bigcap_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{S}_{\mathfrak{n}})$ is non-empty.*

The proof of the following Theorem is similar to proof Theorem 2.6. Kiliçman and Abuage (2015), so omitted.

Theorem 2.1. *A $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is $n\nu$-Lindelöf if and only if each family of ν-regular closed sets of Λ_ν with a countable intersection property admits a non-empty intersection.*

Lemma 2.1. Csàszàr (2008). *Let a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ then*

1. *If $\mathcal{F}$ is ν-closed set then $i_\nu(\mathcal{F})$ is ν-regular open.*
2. *If $\mathcal{U}$ is ν-open set then $c_\nu(\mathcal{U})$ is ν-regular closed.*

Corollary 2.1. *A $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is $n\nu$-Lindelöf if and only if $(\mathcal{X}_{\mathcal{G}}, \nu_\delta)$ is ν_δ-Lindelöf.*

Theorem 2.2. *If a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is G-semiregular and $n\nu$-Lindelöf space then it is ν-Lindelöf.*

Proof. Suppose $\mathfrak{U} = \{\mathcal{U}_\gamma : \gamma \in \Omega\}$ is a ν-open cover of Λ_ν. For each $t \in \Lambda_\nu$, there is $\gamma_t \in \Omega$ such that $t \in \mathcal{U}_{\gamma_t}$. By hypothesis since $\mathcal{U}_{\gamma_t} \in \mathfrak{U}$, there is ν-open set $\mathcal{V}_{\gamma_t}$ such that $t \in \mathcal{V}_{\gamma_t} \subseteq i_\nu c_\nu \mathcal{V}_{\gamma_t} \cap \Lambda_\nu \subseteq \mathcal{U}_{\gamma_t}$. Thus by Lemma 2.1. $\{i_\nu c_\nu(\mathcal{V}_{\gamma_t}) : t \in \Lambda_\nu\}$ is a ν-regular open cover of Λ_ν. Since a $\mathcal{GTS}$ X_G is $n\nu$-Lindelöf, then there is a countable subset of points $t_1, t_2, ..., t_{\mathfrak{n}}, ...$ of Λ_ν such that $\Lambda_\nu = \bigcup_{\mathfrak{n} \in \mathfrak{N}} (i_\nu(c_\nu(\mathcal{V}_{\gamma_{t_{\mathfrak{n}}}}))) \subseteq \bigcup_{\mathfrak{n} \in \mathfrak{N}} (i_\nu(c_\nu(\mathcal{U}_{\gamma_{t_{\mathfrak{n}}}})))$. This implies that $\{\mathcal{U}_{\gamma_{t_{\mathfrak{n}}}} : \mathfrak{n} \in \mathfrak{N}\}$ is a countable sub-collection of $\mathfrak{U}$, hence that completes the proof.

Remark 2.2. *It well known that in topological spaces every nearly Lindelöf almost regular space is nearly paracompact but in generalized topological spaces is not true, since Arar introduced an important example to show that in $\mathcal{GTS}$ there exists a ν-Lindelöf ν-regular space which is not ν-paracompact (for more details see* Arar (2014)*). By the same example we introduce the following corollary:*

Corollary 2.2. *There exists a $n\nu$-Lindelöf almost G-regular space which is not $n\nu$-paracompact.*

Definition 2.3. Sarsak (2012) *Let $(\mathcal{X}_{\mathcal{G}}, \nu)$ and $\mathcal{S} \subseteq \mathcal{X}_{\mathcal{G}}$. Then a collection $\{\mathcal{U} \cap \mathcal{S} : \mathcal{U} \in \nu\}$ is said to be generalized topology on $\mathcal{S}$, and denote by $\nu(\mathcal{S})$. A $\mathcal{GT}$ $\nu(\mathcal{S})$ on $\mathcal{S}$ forms a generalized topological subspace of $\mathcal{X}_{\mathcal{G}}$, denoted by $(\mathcal{S}, \nu(\mathcal{S}))$.*

Definition 2.4. *A subset $\mathcal{S}$ of a ν-space $(\mathcal{X}_{\mathcal{G}}, \nu)$ is said to be*

1. *$n\nu(\mathcal{S})$-Lindelöf if for any $\nu(\mathcal{S})$-regular open cover of $\mathcal{S} \cap \Lambda_\nu$ has a countable sub-collection,*
2. *$n\nu$-Lindelöf relative to $\mathcal{X}_{\mathcal{G}}$ if for each ν-regular open cover $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ of Λ_ν such that $\mathcal{S} \cap \Lambda_\nu \subseteq \bigcup_{\gamma \in \Omega} (\mathcal{U}_\gamma)$, there exists a countable sub-collection $\{\mathcal{U}_{\gamma_{\mathfrak{n}}} : \mathfrak{n} \in \mathfrak{N}\}$ such that*

$$\mathcal{S} \cap \Lambda_\nu \subseteq \bigcup_{\mathfrak{n} \in \mathfrak{N}} (\mathcal{U}_{\gamma_{\mathfrak{n}}})$$

Theorem 2.3. Kiliçman and Abuage (2015)*. Assume $\mathcal{S}$ be a subset of $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ if $\mathcal{S}$ is ν-open then $\nu(\mathcal{S})$-regular open (resp. $\nu(\mathcal{S})$-regular closed) sets in the induced generalized topological subspace $(S, \nu(\mathcal{S}))$ are of the form $\mathcal{S} \cap \mathcal{U}$ where $\mathcal{U}$ is ν-regular open (resp. ν-regular closed) in $\mathcal{X}_{\mathcal{G}}$.*

The following Theorems give some characterization of subsets of $n\nu$-Lindelöf space:

Theorem 2.4. *Let $\mathcal{S}$ is ν-open subset of $n\nu$-Lindelöf $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$, then $\mathcal{S}$ is $n\nu(\mathcal{S})$-Lindelöf if and only if it is $n\nu$-Lindelöf relative to $\mathcal{X}_{\mathcal{G}}$.*

Proof. $(\Rightarrow)$ Impose $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ be ν-regular open cover of Λ_ν such that $\mathcal{S} \cap \Lambda_\nu \subseteq \bigcup_{\gamma\in\Omega}(\mathcal{U}_\gamma)$. Consider $\mathcal{V}_\gamma = \mathcal{S} \cap \mathcal{U}_\gamma$ for each $\gamma \in \Omega$, then by Theorem 2.3, $\{\mathcal{V}_\gamma : \gamma \in \Omega\}$ is $\nu(\mathcal{S})$-regular open cover of $\mathcal{S}$. Thus, there is a countable sub-collection $\{\mathcal{V}_{\gamma_\mathfrak{n}} : \mathfrak{n} \in \mathfrak{N}\}$ such that $\mathcal{S} \cap \Lambda_\nu = \bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{V}_{\gamma_\mathfrak{n}})$. Since $\mathcal{V}_{\gamma_\mathfrak{n}} \subseteq \mathcal{U}_{\gamma_\mathfrak{n}}$ for each $\mathfrak{n} \in \mathfrak{N}$, this implies that $\mathcal{S} \cap \Lambda_\nu \subseteq \bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{U}_{\gamma_\mathfrak{n}})$. Then $\mathcal{S}$ is $n\nu$-Lindelöf relative to $\mathcal{X}_{\mathcal{G}}$.

$(\Leftarrow)$ Let $\{\mathcal{V}_\gamma : \gamma \in \Omega\}$ be $\nu(\mathcal{S})$-regular open cover of $\mathcal{S}$, then by Theorem 2.3, for each $\gamma \in \Omega$, $\mathcal{V}_\gamma = \mathcal{U}_\gamma \cap \mathcal{S}$ where $\mathcal{U}_\gamma$ is ν-regular open, and $\mathcal{S} \subseteq \bigcup_{\gamma\in\Omega}(\mathcal{U}_\gamma)$. Thus, there is a countable sub-collection $\{\mathcal{U}_{\gamma_\mathfrak{n}} : \mathfrak{n} \in \mathfrak{N}\}$ of $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ such that $\mathcal{S} \cap \Lambda_\nu \subseteq \bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{U}_{\gamma_\mathfrak{n}})$, then

$$\mathcal{S} \cap \Lambda_\nu \subseteq \bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{U}_{\gamma_\mathfrak{n}}) \cap \mathcal{S} = \bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{U}_{\gamma_\mathfrak{n}} \cap \mathcal{S}) = \bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{V}_{\gamma_\mathfrak{n}}).$$

That implies a subset $\mathcal{S}$ is $n\nu(\mathcal{S})$-Lindelöf.

Corollary 2.3.

1. *Let $\mathcal{S}$ is ν-regular closed (resp. ν-clopen) subset of $n\nu$-Lindelöf $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ then $\mathcal{S}$ is $n\nu$-Lindelöf relative to $\mathcal{X}_{\mathcal{G}}$.*
2. *Let a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ be a ν-extremally disconnected and $\mathcal{S}$ be a subset of $\mathcal{X}_{\mathcal{G}}$. If $\mathcal{S}$ is $n\nu(\mathcal{S})$-Lindelöf then it is $n\nu$-Lindelöf relative to $\mathcal{X}_{\mathcal{G}}$.*

Definition 2.5. *A $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is said to be $n\nu$-normal if for each ν-regular closed sets $\mathcal{F}_1$ and $\mathcal{F}_2$ with $\mathcal{F}_1 \cap \mathcal{F}_2 = \emptyset$, there are disjoint ν-open sets $\mathcal{U}, \mathcal{V}$ such that $\mathcal{F}_1 \cap \Lambda_\nu \subseteq \mathcal{U}$, $\mathcal{F}_2 \cap \Lambda_\nu \subseteq \mathcal{V}$.*

Theorem 2.5. *Every $n\nu$-Lindelöf almost G-regular space is $n\nu$-normal.*

Proof. Let a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ be a $n\nu$-Lindelöf almost G-regular and $\mathcal{F}_1, \mathcal{F}_2$ be disjoint ν-regular closed sets in $\mathcal{X}_{\mathcal{G}}$. For each $t \in \mathcal{F}_1$, let $\mathcal{G}_t$ be a ν-open set containing t such that $c_\nu(\mathcal{G}_t) \cap (\mathcal{F}_2 \cap \Lambda_\nu) = \emptyset$, by almost G-regularity. Similarly, find a ν-open set $\mathcal{H}_t$ for each $t \in \mathcal{F}_2$ separating t from $\mathcal{F}_2 \cap \Lambda_\nu$. Since $\mathcal{F}_1, \mathcal{F}_2$ are $n\nu$-Lindelöf subspaces of $\mathcal{X}_{\mathcal{G}}$, apparently a countable numbers of the sets $\mathcal{G}_y$ cover $\mathcal{F}_1 \cap \Lambda_\nu$, say $\mathcal{F}_1 \cap \Lambda_\nu \subseteq \bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{G}_\mathfrak{n})$, similarly, $\mathcal{F}_2 \cap \Lambda_\nu \subseteq \bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{H}_\mathfrak{n})$. Now construct ν-open sets $\mathcal{U}_\mathfrak{n}$ and $\mathcal{V}_\mathfrak{n}$ inductively as follows:

$$\begin{array}{cc} \mathcal{U}_1 = \mathcal{G}_1 & \mathcal{V}_1 = \mathcal{H}_1 \backslash c_\nu(\mathcal{U}_1) \\ \mathcal{U}_2 = \mathcal{G}_2 \backslash c_\nu(\mathcal{V}_1) & \mathcal{V}_2 = \mathcal{H}_2 \backslash c_\nu(\mathcal{U}_1 \cup \mathcal{U}_2) \\ \mathcal{U}_3 = \mathcal{G}_3 \backslash c_\nu(\mathcal{V}_1 \cup \mathcal{V}_2) & \mathcal{V}_3 = \mathcal{H}_3 \backslash c_\nu(\mathcal{U}_1 \cup \mathcal{U}_2 \cup \mathcal{U}_3) \\ . & . \\ . & . \\ . & . \end{array}$$

So, it is obviously seen that $\mathcal{U} = \bigcup_{\mathfrak{n} \in \mathfrak{N}} (\mathcal{U}_\mathfrak{n})$ and $\mathcal{V} = \bigcup_{\mathfrak{n} \in \mathfrak{N}} (\mathcal{V}_\mathfrak{n})$ are disjoint ν-open sets containing $\mathcal{F}_1 \cap \Lambda_\nu$ and $\mathcal{F}_2 \cap \Lambda_\nu$, respectively.

3. Mapping Properties

The notions of continuous functions in generalized topological spaces was introduced by Csàszàr (2002). Let ν and μ be generalized topologies on $\mathcal{X}_\mathcal{G}$ and $\mathcal{Y}_\mathcal{G}$, respectively. Then a function $g : (\mathcal{X}_\mathcal{G}, \nu) \to (\mathcal{Y}_\mathcal{G}, \mu)$ from a ν-space $(\mathcal{X}_\mathcal{G}, \nu)$ into a μ-space $(\mathcal{Y}_\mathcal{G}, \mu)$ is called (ν, μ)- continuous iff $\mathcal{U} \in \mu$ implies that $g^{-1}(\mathcal{U}) \in \nu$.

Definition 3.1. *A function $g : (\mathcal{X}_\mathcal{G}, \nu) \to (\mathcal{Y}_\mathcal{G}, \mu)$ is said to be:*

1. *strongly $\theta(\nu, \mu)$- continuous* Min and Kim (2011),
2. *super (ν, μ)- continuous* Min and Kim (2011),
3. *(δ, δ')- continuous* Min (2010a),
4. *almost (ν, μ)- continuous* Min (2009),

if for each $t \in \mathcal{X}_\mathcal{G}$ and each μ-open set $\mathcal{U}$ containing $g(t)$, there is a ν-open set $\mathcal{V}$ containing t such that

1. *$g(c_\nu(\mathcal{V})) \subseteq \mathcal{U}$.*
2. *$g(i_\nu c_\nu(\mathcal{V})) \subseteq \mathcal{U}$.*
3. *$g(i_\nu c_\nu(\mathcal{V})) \subseteq i_\mu c_\mu(\mathcal{U})$.*
4. *$g(\mathcal{V}) \subseteq i_\mu c_\mu(\mathcal{U})$. respectively.*

Remark 3.1. *From the definition above we obtain on the implications below but in general, the converse not be true* (see Min (2010a), Min (2009) and Min and Kim (2011)*).*

$$\begin{array}{ccc} & \textit{strongly } \theta(\nu,\mu)-\textit{continuous} & \\ & \downarrow & \\ & \textit{super}(\nu,\mu)-\textit{continuous} & \\ & \swarrow \quad \searrow & \\ (\nu,\mu)-\textit{continuous} & & (\delta,\delta')-\textit{continuous} \\ & \searrow \quad \swarrow & \\ & \textit{almost}(\nu,\mu)-\textit{continuous} & \end{array}$$

Theorem 3.1. *Let $g:(\mathcal{X}_{\mathcal{G}},\nu)\to(\mathcal{Y}_{\mathcal{G}},\mu)$ be a (δ,δ')- continuous surjection, if a ν-space $\mathcal{X}_{\mathcal{G}}$ is $n\nu$-Lindelöf then so is a μ-space $\mathcal{Y}_{\mathcal{G}}$.*

Proof. Suppose a μ-regular open cover $\{\mathcal{U}_\gamma : \gamma\in\Omega\}$ of a μ-space $\mathcal{Y}_{\mathcal{G}}$, let for each $t\in\mathcal{Y}_{\mathcal{G}}$, each μ-regular open set $\mathcal{U}_{\gamma_t}$ containing $g(t)$. Since g is (δ,δ')-continuous, there is a ν-regular open set $\mathcal{V}_{\gamma_t}$ of $\mathcal{X}_{\mathcal{G}}$ containing t such that $g(\mathcal{V}_{\gamma_t})\subseteq\mathcal{U}_{\gamma_t}$. So, $\{\mathcal{V}_{\gamma_t} : t\in\mathcal{Y}_{\mathcal{G}}\}$ is a ν-regular open cover of $\mathcal{X}_{\mathcal{G}}$. Then there exists a countable ν- sub-collection $\{\mathcal{V}_{\gamma_{t_\mathfrak{n}}} : \mathfrak{n}\in\mathfrak{N}\}$ such that $\mathcal{Y}_{\mathcal{G}}=\bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{V}_{\gamma_{t_\mathfrak{n}}})$. So,

$$\mathcal{Y}_{\mathcal{G}}=g(\mathcal{X}_{\mathcal{G}})=g(\bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{V}_{\gamma_{t_\mathfrak{n}}}))=\bigcup_{\mathfrak{n}\in\mathfrak{N}}g(\mathcal{V}_{\gamma_{t_\mathfrak{n}}})\subseteq\bigcup_{\mathfrak{n}\in\mathfrak{N}}(\mathcal{U}_{\gamma_{t_\mathfrak{n}}}).$$

Thus μ-space $\mathcal{Y}_{\mathcal{G}}$ is $n\mu$-Lindelöf.

On using Remark 3.1 and Theorem 3.1, we got the next result:

Corollary 3.1. *Let $g:(\mathcal{X}_{\mathcal{G}},\nu)\to(\mathcal{Y}_{\mathcal{G}},\mu)$ be a strongly $\theta(\nu,\mu)$-continuous (resp. super (ν,μ)- continuous) surjection, if a ν-space $\mathcal{X}_{\mathcal{G}}$ is $n\nu$-Lindelöf then so is a μ-space $\mathcal{Y}_{\mathcal{G}}$.*

Min and Kim (2011) and Min (2010a) showed that, let $g:(\mathcal{X}_{\mathcal{G}},\nu)\to(\mathcal{Y}_{\mathcal{G}},\mu)$ be a (ν,μ)- continuous (resp. almost (ν,μ)- continuous) function and $\mathcal{X}_{\mathcal{G}}$ is a ν-regular space, hence g is super (ν,μ)- continuous (resp. (δ,δ')- continuous). So, on using Corollary 3.1 and Theorem 3.1 we induced the corollary below:

Corollary 3.2. *Let $g:(\mathcal{X}_{\mathcal{G}},\nu)\to(\mathcal{Y}_{\mathcal{G}},\mu)$ be a (ν,μ)- continuous (resp. almost (ν,μ)- continuous) surjection and $\mathcal{X}_{\mathcal{G}}$ is ν-regular ν-space, if a ν-space $\mathcal{X}_{\mathcal{G}}$ is $n\nu$-Lindelöf space so is a μ-space $\mathcal{Y}_{\mathcal{G}}$.*

Definition 3.2. *A function $g:(\mathcal{X}_{\mathcal{G}},\nu)\to(\mathcal{Y}_{\mathcal{G}},\mu)$ is called almost (ν,μ)-open* Al-Omari and Noiri (2012) *(resp. weakly (ν,μ)-open) if $g(\mathcal{V})\subseteq i_\mu c_\mu(g(\mathcal{V}))$ (resp. $g(\mathcal{V})\subseteq i_\mu(g(c_\nu(\mathcal{V})))$, for any ν-open set $\mathcal{V}$ in $\mathcal{X}_{\mathcal{G}}$.*

Obviously, every almost (ν,μ)-open function is weakly (ν,μ)-open but the converse is not true in general as an example shows:

Example 3.1. *Let $\mathcal{X}_{\mathcal{G}} = \{a,b,c\}$, $\nu = \{\emptyset, \{a\}, \{b\}, \{a,b\}, Y\}$ and $\mu = \{\emptyset, \{c\}, \{a,c\}, \{b,c\}, \mathcal{X}_{\mathcal{G}}\}$. Then the identity function $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ is weakly (ν,μ)-open. However, there is a ν-open set $\{a\}$ in $\mathcal{X}_{\mathcal{G}}$ such that $g(\{a\}) = \{a\}$ not contained in $i_\mu c_\mu(g(\{a\})) = \emptyset$, so a function g is not an almost (ν,μ)-open.*

Theorem 3.2. *Every weakly (ν,μ)-open almost (ν,μ)- continuous function is (δ,δ')- continuous.*

Proof. Let $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ be a weakly (ν,μ)-open almost (ν,μ)- continuous function. Suppose $\mathcal{U}$ be a μ-regular open set in $\mathcal{Y}_{\mathcal{G}}$. Since a function g is an almost (ν,μ)- continuous, $g^{-1}(\mathcal{U})$ is ν-open in $\mathcal{X}_{\mathcal{G}}$. So $g^{-1}(\mathcal{U}) \subseteq i_\nu c_\nu(g^{-1}(\mathcal{U}))$. Now we have to show the opposite inclusion. Since g is a weakly (ν,μ)-open and $i_\nu c_\nu(g^{-1}(\mathcal{U}))$ is ν-open set in $\mathcal{X}_{\mathcal{G}}$,

$$g(i_\nu c_\nu(g^{-1}(\mathcal{U}))) \subseteq i_\mu(g(c_\nu(i_\nu(c_\nu(g^{-1}(\mathcal{U})))))) \subseteq i_\mu(g(c_\nu(g^{-1}(\mathcal{U})))).$$

Since g is an almost (ν,μ)- continuous and $\mathcal{U}$ be a μ-regular open set, then $c_\mu(\mathcal{U})$ is μ-regular closed in a μ-space $\mathcal{Y}_{\mathcal{G}}$. Thus by Theorem 3.6 (7) Min (2009), $g^{-1}(c_\mu(\mathcal{U}))$ is μ-closed set in ν-space $\mathcal{X}_{\mathcal{G}}$ and hence $c_\nu(g^{-1}(\mathcal{U})) \subseteq c_\nu(g^{-1}(c_\mu(\mathcal{U}))) = g^{-1}(c_\mu(\mathcal{U}))$. So,

$$g(i_\nu c_\nu(g^{-1}(\mathcal{U}))) \subseteq i_\mu(g(g^{-1}(c_\nu(\mathcal{U})))) \subseteq i_\mu c_\mu(\mathcal{U}) = \mathcal{U}.$$

Thus $i_\nu c_\nu(g^{-1}(\mathcal{U})) \subseteq g^{-1}(\mathcal{U})$, hence $i_\nu c_\nu(g^{-1}(\mathcal{U})) = g^{-1}(\mathcal{U})$. Which implies that $g^{-1}(\mathcal{U})$ is ν-regular open, i.e. g is (δ,δ')- continuous and this completes the proof.

Corollary 3.3. *Every almost (ν,μ)-open almost (ν,μ)- continuous function is (δ,δ')- continuous.*

Through the Theorem 3.2, Theorem 3.1 and Corollary 3.3, we conclude the following result:

Corollary 3.4. *Let $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ be a weakly (ν,μ)-open (resp. almost (ν,μ)-open) and almost (ν,μ)- continuous surjection, if a ν-space $\mathcal{X}_{\mathcal{G}}$ is $n\nu$-Lindelöf space then so is a μ-space $\mathcal{Y}_{\mathcal{G}}$.*

In Theorem 2.2 and Theorem 2.5, it was proved that, if a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is G-semiregular (resp. almost G-regular) and $n\nu$-Lindelöf then it is ν-Lindelöf (resp. $n\nu$-normal). Thus we obtained on the next corollary:

Corollary 3.5. *Let $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ be a function from a ν-regular ν-space $\mathcal{X}_{\mathcal{G}}$ onto a G-semiregular (resp. almost G-regular) ν-space $\mathcal{Y}_{\mathcal{G}}$ satisfying one of the following conditions:*

1. *(δ, δ')- continuous,*
2. *strongly $\theta(\nu, \mu)$- continuous (resp. super (ν, μ)- continuous),*
3. *(ν, μ)- continuous (resp. almost (ν, μ)- continuous),*
4. *weakly (ν, μ)-open (resp. almost (ν, μ)-open) and almost (ν, μ)- continuous function.*

If a ν-space $\mathcal{X}_{\mathcal{G}}$ is $n\nu$-Lindelöf then a μ-space $\mathcal{Y}_{\mathcal{G}}$ is μ-Lindelof (resp. $n\mu$-normal).

Definition 3.3. *Let $\mathcal{S}$ be a subset of $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$, then $\mathcal{S}$ is called ν-preopen (resp. $\nu - \beta$-open)* Csàszàr (2005) *if $\mathcal{S} \subseteq i_\nu c_\nu(\mathcal{S})$ (resp. $\mathcal{S} \subseteq c_\nu i_\nu c_\nu(\mathcal{S})$).*

We denote by π the class of all ν-preopen sets in $\mathcal{X}_{\mathcal{G}}$, by β the class of all $\nu - \beta$-open sets in $\mathcal{X}_{\mathcal{G}}$.

Definition 3.4. *A function $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ is said to be*

1. *(π, μ)- continuous (resp. (β, μ)- continuous)* Min (2010b) *if*
$$g^{-1}(\mathcal{U}) \subseteq i_\nu c_\nu(g^{-1}(\mathcal{U}))$$
(resp. $g^{-1}(\mathcal{U}) \subseteq c_\nu i_\nu c_\nu(g^{-1}(\mathcal{U}))$) for every μ-open set $\mathcal{U}$.
2. *almost (π, μ)- continuous (resp. almost (β, μ)- continuous) if for each $t \in \mathcal{X}_{\mathcal{G}}$ and each μ-regular open set $\mathcal{U}$ in a μ-space $\mathcal{Y}_{\mathcal{G}}$ containing $g(t)$, there is ν-preopen (resp. $\nu - \beta$-open) set $\mathcal{V}$ containing t such that $g(\mathcal{V}) \subseteq \mathcal{U}$.*

Remark 3.2. *Let $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ be a function between $\mathcal{GTS}'s$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ and $(\mathcal{Y}_{\mathcal{G}}, \mu)$. Then we have the following implications but the reverse relations may not be true in general:*

almost (ν, μ)- continuous $\Rightarrow$ almost (π, μ)- continuous $\Rightarrow$ almost (β, μ)- continuous.

Example 3.2. *Let* $\mathcal{X}_{\mathcal{G}} = \{x, y, z\}$

1. *if* $\nu = \{\emptyset, \{x, y\}\}$ *be a* $\mathcal{GT}$ *on* $\mathcal{X}_{\mathcal{G}}$. *Then* $\pi = \nu \cup \{\{x\}, \{y\}\}$. *Define a function* $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{X}_{\mathcal{G}}, \nu)$ *as follows:* $g(x) = x$, $g(y) = g(z) = z$. *Then* g *is* (π, ν)*- continuous function but not almost* (ν, μ)*- continuous.*

2. *If* $\nu = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$ *be a GT on* $\mathcal{X}_{\mathcal{G}}$. *Then* $\pi = \nu$ *and* $\beta = \nu \cup \{\{x, y\}, \{x, z\}, \mathcal{X}_{\mathcal{G}}\}$. *Consider a function* $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{X}_{\mathcal{G}}, \nu)$ *defined by* $g(x) = g(y) = y$, $g(z) = x$. *Then* g *is almost* (β, ν)*- continuous function without begin* (π, ν)*- continuous.*

Obviously, if $\mathcal{X}_{\mathcal{G}} \in \nu$ in a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ then $c_\nu(\emptyset) = \emptyset$, so the following theorem proves immediately by Theorem 30. Ekici (2012), so the proof omitted.

Theorem 3.3. *Let* $(\mathcal{X}_{\mathcal{G}}, \nu)$ *be a submaximal and* ν*-extremally disconnected* ν*-space. Then a function* $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ *is an almost* (ν, μ)*- continuous if and only if it is almost* (β, μ)*- continuous.*

On using Theorem 3.3 and Corollary 3.4, we induced the following corollary:

Corollary 3.6. *Let* $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ *be a weakly* (ν, μ)*-open (resp. almost* (ν, μ)*-open) and almost* (β, μ)*- continuous surjection. If* $(\mathcal{X}_{\mathcal{G}}, \nu)$ *is submaximal,* ν*-extremally disconnected and* $n\nu$*-Lindelöf* ν*-space then a* μ*-space* $\mathcal{Y}_{\mathcal{G}}$ *is* $n\mu$*-Lindelöf.*

Lemma 3.1. *Let* $(\mathcal{X}_{\mathcal{G}}, \nu)$ *be a submaximal* $\mathcal{QTS}$ *then every* ν*-preopen set is* ν*-open.*

Proof. Assume, a subset $\mathcal{V}$ is a ν-preopen, then by Proposition 3.11 Sarsak (2013) $\mathcal{V} = \mathcal{U} \cap \mathcal{S}$ for some ν-regular open set $\mathcal{U}$ and ν-dense set $\mathcal{S}$ of $\mathcal{X}_{\mathcal{G}}$. Since $(\mathcal{X}_{\mathcal{G}}, \nu)$ is submaximal $\mathcal{QTS}$, so $\mathcal{S}$ is ν-open set of $\mathcal{X}_{\mathcal{G}}$ and thus $\mathcal{V}$ is ν-open set of $\mathcal{X}_{\mathcal{G}}$.

Next Theorem proves directly, by Lemma 3.1, so omitted.

Theorem 3.4. *Let* $(\mathcal{X}_{\mathcal{G}}, \nu)$ *be a submaximal* $\mathcal{QTS}$ *then a function* $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ *is an almost* (ν, μ)*- continuous if and only if it is almost* (π, μ)*- continuous.*

Since every ν-space under closed intersection is topological space, thus by Theorem 3.4 and Corollary 3.4, the following corollary concluded:

Corollary 3.7. *Let $g : (\mathcal{X}_{\mathcal{G}}, \tau) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ be an almost (π, μ)- continuous and weakly (τ, μ)-open (resp. almost (τ, μ)-open) surjection. If a space $(\mathcal{X}_{\mathcal{G}}, \tau)$ is submaximal and nearly Lindelöf then a μ-space $\mathcal{Y}_{\mathcal{G}}$ is $n\mu$-Lindelöf.*

Theorem 3.5. *Let $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu_\delta)$ be (π, μ_δ)- continuous (resp. (β, μ_δ)- continuous) function then $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ is an almost (π, μ)- continuous (resp. almost (β, μ)- continuous).*

Proof. Obviously, where every μ-regular open set in $\mathcal{Y}_{\mathcal{G}}$ is μ_δ-open.

In Corollary 2.1 it was indicated that a $\mathcal{GTS}$ $(\mathcal{X}_{\mathcal{G}}, \nu)$ is $n\nu$-Lindelöf if and only if (Y, ν_δ) is ν_δ-Lindelöf. On using Theorem 3.5, Corollary 3.6 and Corollary 3.7, we conclude the corollaries below:

Corollary 3.8. *Let $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu_\delta)$ is (π, μ_δ)- continuous (resp. (β, μ_δ)- continuous) and weakly (ν, μ)-open (resp. almost (ν, μ)-open) surjection. If $(\mathcal{X}_{\mathcal{G}}, \nu)$ is submaximal, ν-extremally disconnected and $n\nu$-Lindelöf ν-space then a μ-space $(\mathcal{Y}_{\mathcal{G}}, \mu_\delta)$ is μ-Lindelöf.*

Corollary 3.9. *Let $g : (\mathcal{X}_{\mathcal{G}}, \tau) \to (\mathcal{Y}_{\mathcal{G}}, \mu_\delta)$ is (π, μ_δ)- continuous (resp. (β, μ_δ)- continuous) and weakly (τ, μ)-open (resp. almost (τ, μ)-open) surjection. If $(\mathcal{X}_{\mathcal{G}}, \tau)$ is submaximal and nearly Lindelöf space then a μ-space $(\mathcal{Y}_{\mathcal{G}}, \mu_\delta)$ is μ-Lindelöf.*

Theorem 3.6. *Let a μ-space $(\mathcal{Y}_{\mathcal{G}}, \mu)$ be a G-semiregular, then a function $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ is (π, μ_δ)- continuous (resp. (β, μ_δ)- continuous) if and only if g is almost (π, μ)- continuous (resp. almost (β, μ)- continuous).*

Proof. $(\Rightarrow)$ Obviously, so omitted.

$(\Leftarrow)$ Suppose, for each $t \in \mathcal{X}_{\mathcal{G}}$ and each μ_δ-open set $\mathcal{U}$ of μ-space $\mathcal{Y}_{\mathcal{G}}$ with $g(t) \in \mathcal{U}$. By G-semiregularity of μ-space $\mathcal{Y}_{\mathcal{G}}$, $\mathcal{U}$ is μ-open in $\mathcal{Y}_{\mathcal{G}}$ containing $g(t)$ and hence there is μ-open set $\mathcal{O}$ of $\mathcal{Y}_{\mathcal{G}}$ such that $g(t) \in \mathcal{O} \subseteq i_\mu c_\mu(\mathcal{O}) \subseteq \mathcal{U}$. Since g is an almost (π, μ)- continuous (resp. almost (β, μ)- continuous) function, then there is ν-preopen (resp. $\nu - \beta$-open) set $\mathcal{V}$ containing t such that $g(V) \subseteq i_\mu c_\mu(\mathcal{O})$, and this is implies $g(V) \subseteq \mathcal{U}$. So g is (π, μ_δ)- continuous (resp. (β, μ_δ)- continuous) function.

By Theorem 3.6, Corollary 3.6, Corollary 3.7 and Theorem 2.2, we conclude the following corollaries, which is stronger than Corollaries 3.8 and 3.9, respectively.

Corollary 3.10. *Let a μ-space $(\mathcal{Y}_{\mathcal{G}}, \mu)$ be a G-semiregular and let $g : (\mathcal{X}_{\mathcal{G}}, \nu) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ is (π, μ)- continuous (resp. (β, μ)- continuous) and weakly (ν, μ)-open (resp. almost (ν, μ)-open) surjection. If $(\mathcal{X}_{\mathcal{G}}, \nu)$ is submaximal, ν-extremally disconnected and $n\nu$-Lindelöf ν-space then a μ-space $\mathcal{Y}_{\mathcal{G}}$ is μ-Lindelöf.*

Corollary 3.11. *Let a μ-space $(\mathcal{Y}_{\mathcal{G}}, \mu)$ be a G-semiregular and let $g : (\mathcal{X}_{\mathcal{G}}, \tau) \to (\mathcal{Y}_{\mathcal{G}}, \mu)$ is (π, μ)- continuous (resp. (β, μ)- continuous) and weakly (τ, μ)-open (resp. almost (τ, μ)-open) surjection. If $(\mathcal{X}_{\mathcal{G}}, \tau)$ is submaximal and nearly Lindelöf space then a μ-space $\mathcal{Y}_{\mathcal{G}}$ is μ-Lindelöf.*

4. Conclusion

In our work we have introduced nearly ν-Lindelöf (briefly. $n\nu$-Lindelöf) space and subspaces in generalized topological spaces, some mappings and decompositions of continuity are studied. The main result that we obtained on is the effect of (δ, δ')-continuous function on $n\nu$-Lindelöf space.

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